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Correlation Between Transverse Momentum and Multiplicity for Spherically Exploding Quark-Gluon Plasmas

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ABSTRACT

We argue that the mean transverse momentum of particles produced in very high energy nucleus-nucleus collisions or in very high multiplicity $\bar{p}p$ collisions may be directly related to the energy per unit entropy of the hot matter originally formed. We also argue that the initial energy density is related to the mean transverse momentum and to the rapidity density of pions. A tentative analysis of cosmic ray data, and of $\bar{p}p$ collider data finds support for the conjecture that the matter produced in such collisions undergoes a phase transition from a hadron gas to a quark-gluon plasma.

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In the collisions of nuclei at ultra-relativistic energies or in $\bar{p}p$ collisions at very high multiplicity, sufficiently high energy density may be deposited in a volume which is large enough that a quark-gluon plasma might be formed. There have been many suggestions concerning possible signals which might probe the plasma. In particular there has been a suggestion by E. Shuryak(1) and by L. van Hove,(2) that if transverse momentum is plotted versus multiplicity, then the effects of a phase transition might appear as a flattening of this curve for a finite range of multiplicity. Such a correlation has indeed been observed in fluctuations in $\bar{p}p$ collisions as well as in ultra-relativistic nuclear collisions observed by the JACEE cosmic ray experiment. In Fig. 1 we plot this curve.(3)

The theoretical underpinnings of the conjecture that this correlation probes the production of a quark-gluon plasma have their origin in the observation that in a first order phase transition both the temperature and pressure of a mixed phase remain constant as matter of one phase is converted into that of another. The energy densities of the two phases are quite different at the critical temperature and pressure, therefore the energy density of the mixed phase varies considerably as hadronic matter is converted into quark-gluon plasma. During the expansion of the thermally excited plasma, internal energy is converted into the collective flow energy due to pressure gradiants at the surface. If the pressure at the center of the matter remains constant for a range of energy densities, the final transverse momentum will also not change much. Since the total hadron multiplicity provides some measure of the energy density achieved in the collision, the qualitative features of the correlation proposed by Shuryak and by van Hove are to be expected.

To provide a more quantitative framework to study this correlation seems at first site complicated. Three dimensional relativistic hydrodynamic equations must be solved to compute the transverse momentum and multiplicity distributions. While such a computation is necessary to provide a detailed description, and although such a computation is straightforward, it is not necessary to understand the essential features of spherically expanding systems.

In certain in high energy $\bar{p}p$ collisions, spherically symmetric fluctuations might be expected. The distinctive feature of such fluctuations is azimuthal symmetry combined with a high multiplicity in a region of one unit of rapidity. Fluctuations which parallel the cylindrical geometry expected of average nucleus-nucleus collisions might be inferred if it is uniform over several units of rapidity. Clustering in less than one unit of rapidity has no obvious direct interpretation, except as the formation of jets, which from this perspective is a background. Although the rise in transverse momentum observed in UA1 might be explained by mini-jets, the rise observed in the JACEE collaboration may be more difficult to explain by this hypothesis.(4) We shall here assume that such a fluctuation is generated by some random statistical process which is not a jet, assume that a quark-gluon plasma forms, and explore the consequences of this assumption.

The simplicity of the spherical expansion arises from the following considerations. In hydrodynamic expansion, the total energy and entropy are conserved. At a first order phase transition some entropy may be produced if the nucleation rate is not fast compared to the expansion rate. Also, when the hadronic matter decouples to form free hadrons, entropy may be produced. We shall ignore such entropy production, which is not expected to be large, and assume that the dynamics of the matter is described by perfect fluid, non-viscous, hydrodynamic equations.

The average transverse momentum per particle is given by the energy per particle, up to a constant, due to the spherical nature of the expansion. Neglecting the pion rest mass we have $E/N = 4p_\perp/\pi$. The entropy for an ideal gas of massless pions is porportional to the total number of particles, $S \sim 3.7N$. This relation is independent of the radial flow velocity. Even at late times in the expansion of the fluid, when the temperature is about 100 Mev, the pions may be treated as massless to a fair approximation. Furthermore, pion number changing processes are essentially zero.[5] Since both the entropy and pion number are conserved from this point on, and since an ideal pion gas approximation should be valid at about this temperature, this relation between pion number and entropy should be valid. We therfore have $P_\perp/N \sim 2.9\, E/S$, <u>and E/S may be evaluated at any time with the same result, but the transverse momentum is that appropriate in the final state.</u>

The quantity E/S can be simply evaluated in terms of the local energy density E/V for matter formed initially in the collision. Initially, the matter as observed in a local comoving frame should have no collective flow. Therefore, we assume that the matter forms a spherical droplet, which may have net longitudinal momentum, but in the frame where this longitudinal momentum is zero, the matter in the droplet is initially at rest. It is in this frame that the droplet appears to be spherical.

The quantities E/S and E/V may be evaluated and related to each other if the equation of state of the produced matter is known. In particular, for a bag model description of the matter, which is in qualitative agreement with lattice gauge theory computations, and with what is known about the properties of hadronic matter, these quantities may be computed.[6] For such a model, the energy density in the hadronic phase is taken to be an ideal pion gas,

$$\varepsilon = g_h\, \pi^2/30\; T^4 \qquad (1)$$

Here g_h is the number of degrees of freedom of the pion gas, $g_h = 3$. For the quark gluon plasma the energy density is

$$\varepsilon = g_p \pi^2/30\ T^4 + B \tag{2}$$

where the number of degrees of freedom of the plasma is $g_p = 42$, where we have counted the strange quarks as contributing 1/2 a degree of freedom at the temperatures of interest. Such an equation of state is the simplest form which extrapolates between an ideal pion gas, which is appropriate at low temperatures and a quark-gluon plasma at high temperatures, with a rapid transition in between. The magnitude of the rapid change in energy density is controlled by the bag constant, which we shall take as

$$B^{1/4} = 200 \text{ Mev} \tag{3}$$

which leads to a critical temperature of $T_c \sim 140$ Mev. This value is somewhat larger than that given by the MIT bag analysis of hadron spectroscopy, but is consistant with lattice thermodynamic computations. The critical temperature is also somewhat smaller than the value abstracted from lattice Monte-Carlo simulations in the absence of fermions, but is consistent with expectations for the range of values which may be true when fermions are properly included, and is tha value of the Hagedorn limiting temperature. In any case its precise numerical value is not so important for our purposes.

In the region of energy density where a rapid transition takes place between a pion gas and a quark-gluon plasma, a mixed phase of constant pressure occurs. (If there is not a first order phase transition but only a rapid change in the energy density accompanied by a slow change in the pressure, these considerations are not altered.) In the mixed phase, some fraction of the matter is a hadron gas with the appropriate energy density and entropy density, and the other fraction is a quark-gluon plasma with a different energy density and entropy density.

At any specified energy density, the functional relationship between E/S and E/V may be computed. Qualitatively, this relationship is determined in the following way. E/S is the energy per degree of freedom, and is approximately the temperature. In the pion gas and in the plasma, the temperature rises with increasing energy density. In the mixed phase, the temperature is constant, and E/S is approximately constant. An explicit computation yields the curve shown in Fig. 2. The width of the mixed phase is found from the energy density region where the temperature is constant, which is the latent heat of a first order phase transition.

In order to make a comparison with experiment, E/V must be determined. Recall that E/N has already been determined in terms of the average transverse momentum. Part of E/V has already been found, namely the total energy as measured in the local rest frame of the matter, by measuring the energies of produced particles. For spherical expansion most of the particles appear in one unit of rapidity, thus $E \sim p_\perp \frac{dN}{dy}$, where $\frac{dN}{dy}$ is the rapidity density of all particles. The spatial volume in which the matter first forms is more difficult to extract. If the high density matter is formed by a fluctuation in the collisions of pointike constituents, and if the probability of overlapping fluctuations is small, it is not unreasonable to assume that $V \sim 1/p_\perp^3$. The constant of proportionality is difficult to extract without a detailed dynamical model, and since we have no such model, it must be determined empirically.

This relation between V and $p_\perp$ is not expected to be true for <u>average</u> nucleus-nucleus collisions of <u>very large</u> nuclei since many overlapping interactions generate high energy density. In such a situation the transverse size is determined by geometry. The longitudinal distance scale in which matter formation takes place is $1/p_\perp$ from uncertainty principle arguments, and the volume scales accordingly. This initial longitudinal distance scale in which matter forms may also be abstracted from integrating the hydrodynamic equations backwards in time until entropy production becomes a dominant effect. At early times, the condition that entropy production be small may be shown to be the initial condition $T \sim 1/t_0$ which therefore relates initial time, distance, and transverse momentum in the way described above. We should note however that a sequence of spherically exploding quark-gluon plasma balls separated in rapidity along the beam axis could be indistinguishable experimentally from cylindrical symmetry.

When comparing the JACEE data, the fluctuaions in $\bar{p}p$ collisions measured at the S$\bar{p}$pS and a theoretical computation where $V \sim 1/p_\perp^3$, many reservations must be stated. In the JACEE data, collisions of many different nuclei. heavy and light, are included. At least for the collisions of heavy nuclei, the volume probably scales as $1/p_\perp$. The expansion of the produced matter is not spherical, and a detailed hydrodynamic computation must be carried out to relate E/V and E/N. For collisions of light nuclei, the relation $E/V \sim 1/p_\perp^3$ may be valid, but without knowledge of the rapidity space structure of the charged particle distribution, the assumption of spherical expansion may be suspect. Again a detailed hydrodynamic computation is again required. For the collisions of nuclei, we use $E/V \sim p_\perp^4 \frac{dN}{dy} A_{min}^{-2/3}$, where A_{min} is the baryon number of the smaller nucleus participating in the collision. Including this factor of A assumes that spherical droplets may form independently over the transverse extent of the nucleus.

Given all the caveats concerning the use of current nucleus-nucleus data, the use of data from $\bar{p}p$ collisions seems welcome. Unfortunately, data at sufficiently high multiplicity does not yet exist to reproduce the enhancement in transverse momentum inferred from the JACEE data. In addition, it is possible that any enhancement in the transverse momentum seen in the S$\bar{p}$pS at CERN may be due to jet production.[7] This may also be true of the nucleus-nucleus data.

Given all of these reservations, a comparison between theory and experiment before data comes out of the Fermilab quark-gluon plasma search seems premature.[8] Perhaps our computations may at best be regarded as a prediction of what may be expected from such experiments if a solid case may be made that a quark-gluon plasma is produced. Nevertheless, we have made a bold comparison between theory and experiment, and for whatever reasons, the agreement is remarkable. In Fig. 3, E/N is plotted against $E/V = 4.7\ p_\perp^4 \frac{dN}{dy} A_{min}^{-2/3}$, where $p_\perp$ is measured in Gev and E/V in Gev/Fm^3. The total charged multiplicity is dN/dy in this formula, and a factor of 3/2 hase been included to convert this to total pion multiplicity. The only free parameter is the constant of proportionality between V and $p_\perp$. Whether this fit is fortuitous, or whether these considerations, naive as thay are, are quantitatively valid must await more data and more detailed hydrodynamic computations.

One check on these considerations follows from an argument of Shuryak[9] and of Siemans and Rasmussen.[10] The $p_\perp$ enhancement shown in Fig. 3 must arise from the collective motion of the matter. If this is so the fluid has a large outward fluid velocity. Massive particles therefore receive a much greater transverse momentum enhancement than do pions. An experimental observation of such an enhancement would strenghten the case that a plasma was formed, and might discriminate against jets.

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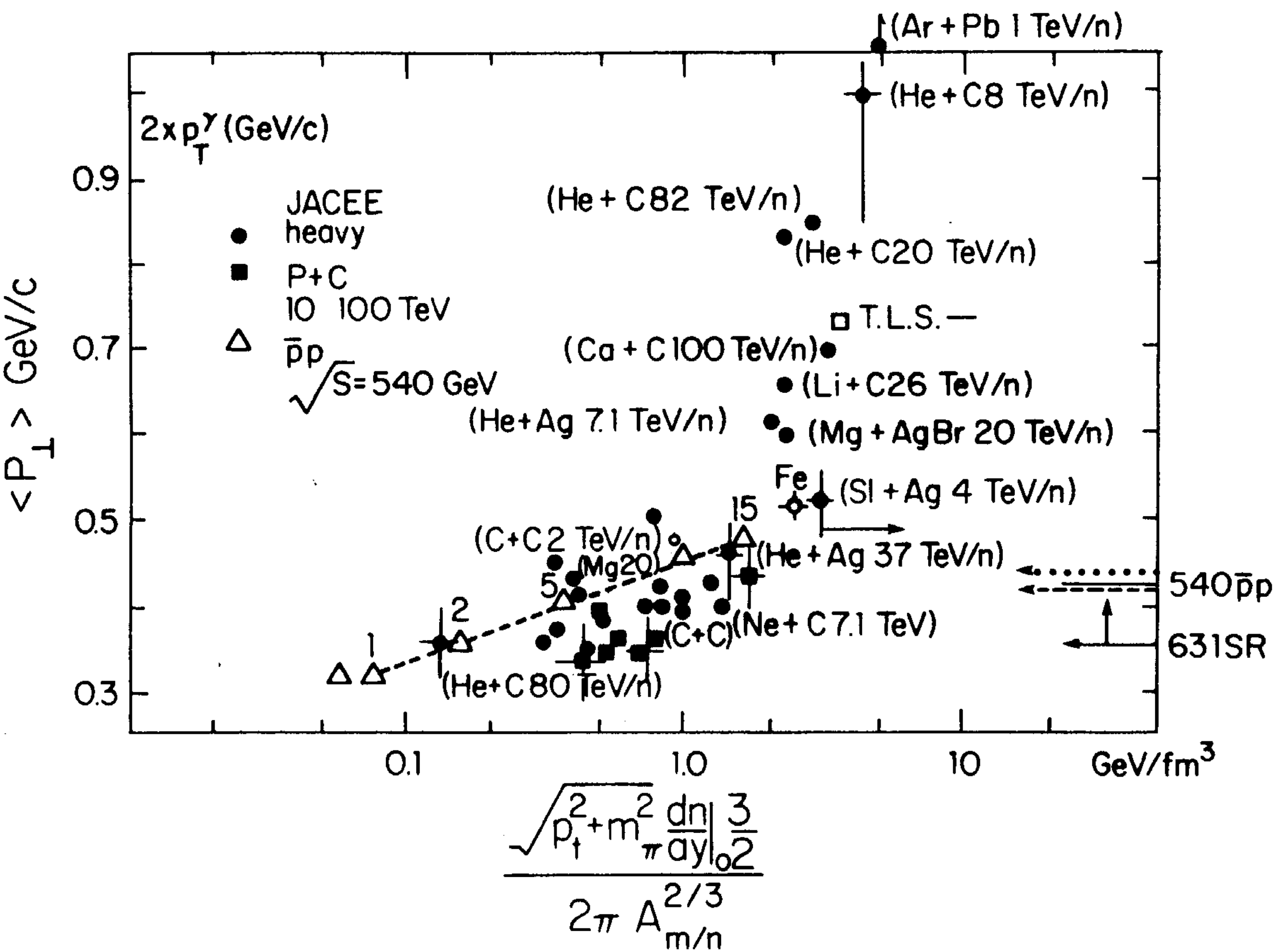
Figure Captions:

Figure 1: A compilation of data for p_t vs dE/dy from Ref. 3.

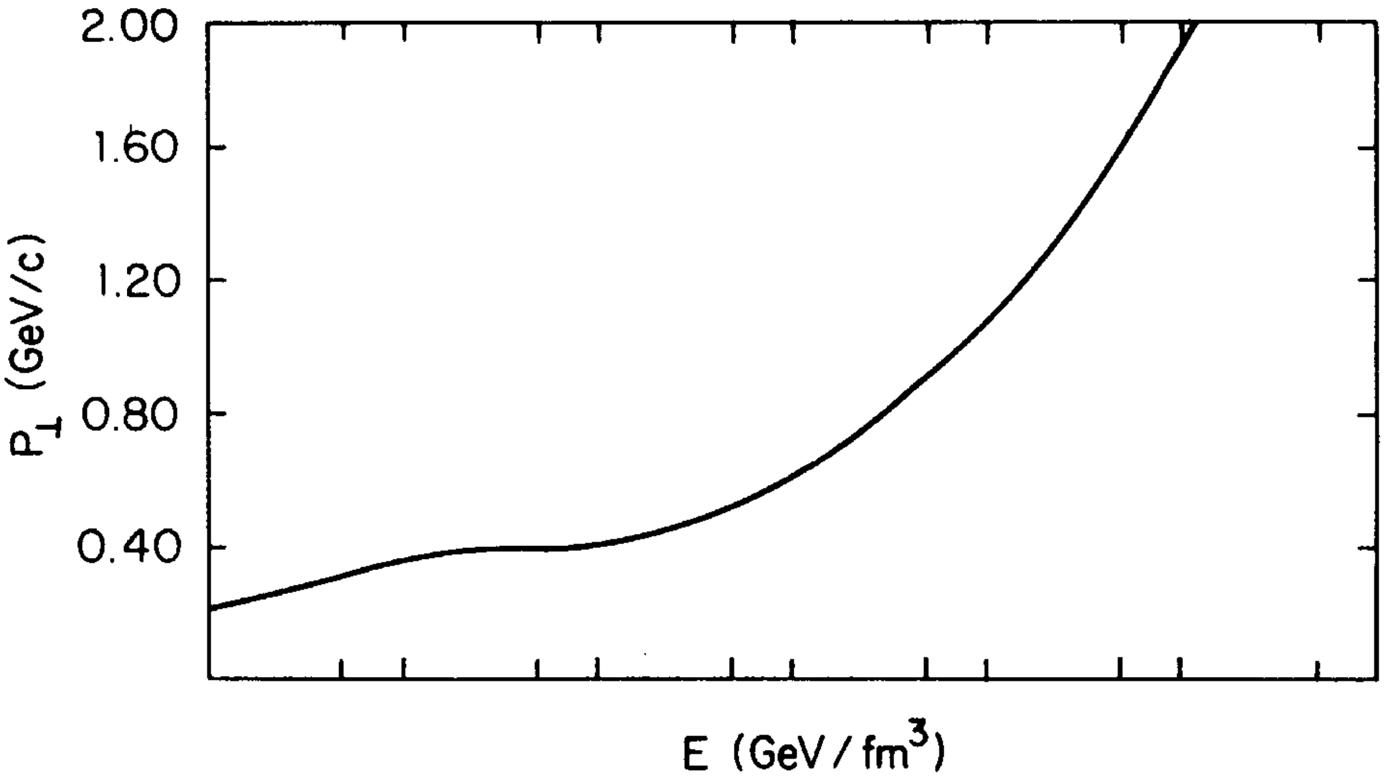
Figure 2: Plot of $p_t = 2.9$ E/S versus energy density E/V for the equation of state described in the text. The flat portion of the curve corresponds to a phase mixture of pion gas and quark-gluon plasma.

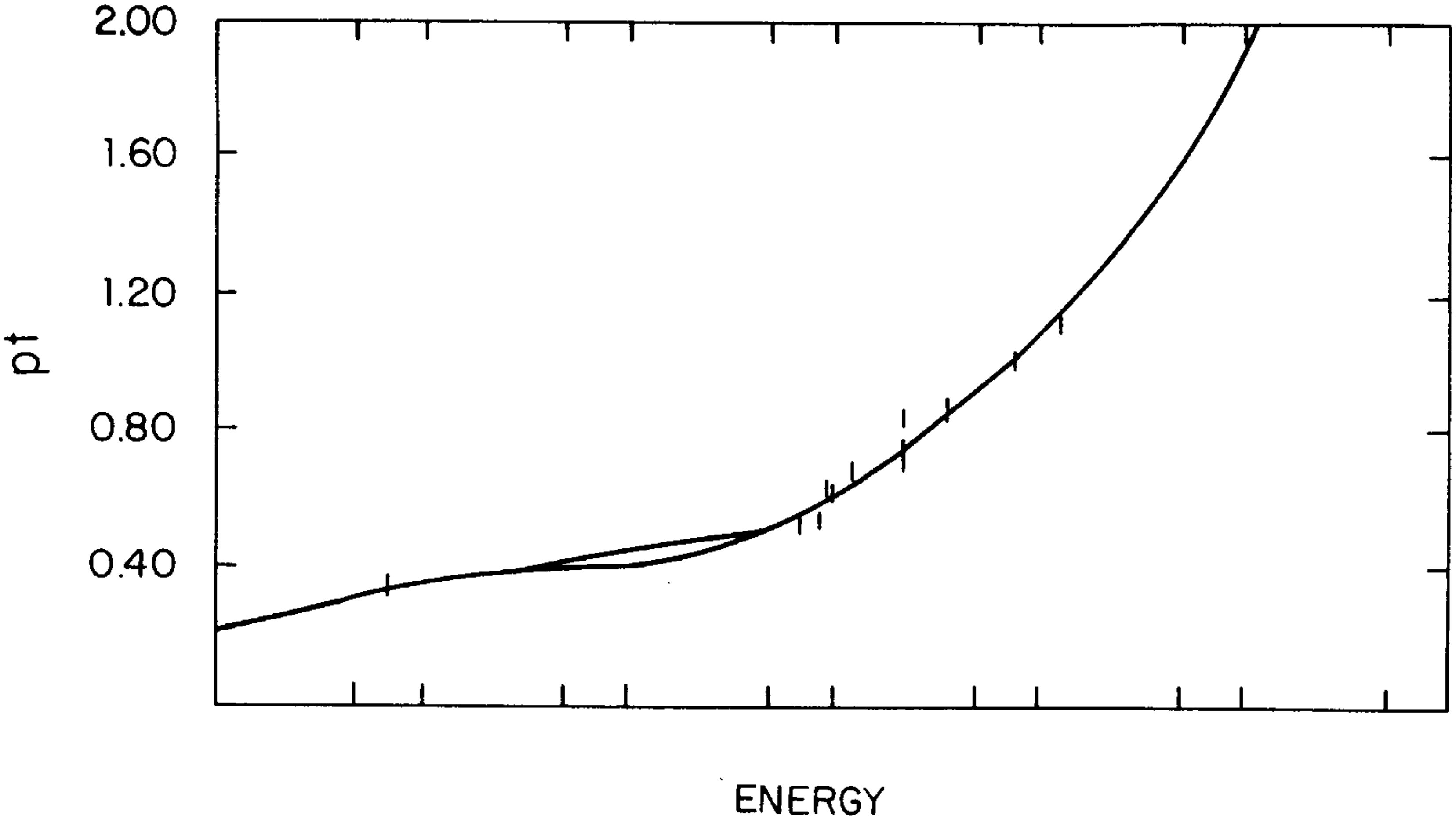
Figure 3: Same theoretical curve as in Fig.2. The data from Fig. 1 have been inserted based on the relation $E/V = 4.7\, p_t^4 \frac{dN}{dy} A^{2/3}_{min}$ with p_t in Gev/c.



2xp_T^γ (GeV/c)
JACEE heavy
P+C 10 100 TeV
p̄p √S=540 GeV
(Ar + Pb 1 TeV/n)
(He + C8 TeV/n)
(He + C82 TeV/n)
(He + C20 TeV/n)
T.L.S.
(Ca + C100 TeV/n)
(Li + C26 TeV/n)
(He+Ag 7.1 TeV/n)
(Mg + AgBr 20 TeV/n)
Fe
(SI + Ag 4 TeV/n)
15
(C+C 2 TeV/n)
(Mg20)
(He + Ag 37 TeV/n)
5
2
1
(C+C)
(Ne + C 7.1 TeV)
(He+C 80 TeV/n)
540p̄p
631SR
0.3
0.5
0.7
0.9
0.1
1.0
10
GeV/fm^3

Fig. 2





ENERGY DENSITY IN THE BAG MODEL, BAG = .200